HYDRO DYNAMICS

FUNDAMENTALS OF FLUID FLOW

Energy and Head of Flow

Energy is defined as ability to do work. Both energy and work are measured in *Newton-meter* (or *pounds-foot* in English). *Kinetic energy* and *potential energy* are the two commonly recognized forms of energy. In a flowing fluid, potential energy may in turn be subdivided into energy due to position or elevation above a given datum, and energy due to pressure in the fluid. *Head* is the amount of *energy per Newton* (or *per pound*) of fluid.

Kinetic Energy and Velocity Head

Kinetic energy is the ability of a mass to do work by virtue of its velocity. The kinetic energy of a mass M having a velocity v is $\frac{1}{2}Mv^2$. Since $M = W/g$,

Fundamentals of Fluid Flow

Energy and Head of Flow

Kinetic Energy and Velocity Head

$$K.E. = W\frac{v^2}{2g}$$

$$\text{Velocity head} = \frac{K.E.}{W} = \frac{v^2}{2g}$$

Velocity Head of Circular Pipes

The velocity head of circular pipe of diameter D flowing full can be found as follows.

$$\frac{v^2}{2g} = \frac{(Q/A)^2}{2g} = \frac{Q^2}{2gA^2}$$

$$\frac{v^2}{2g} = \frac{Q^2}{2g(\frac{1}{4}\pi D^2)^2} = \frac{16Q^2}{2g(\pi^2 D^4}$$

$$\frac{v^2}{2g} = \frac{8Q^2}{\pi^2 g D^4}$$

Fundamentals of Fluid Flow

Energy and Head of Flow

Elevation Energy and Elevation Head

In connection to the action of gravity, elevation energy is manifested in a fluid by virtue of its position or elevation with respect to a horizontal datum plane.

$$\text{Elevation energy} = Wz$$

$$\text{Elevation head} = \frac{\text{Elevation energy}}{W} = z$$

Fundamentals of Fluid Flow

Energy and Head of Flow

Pressure Energy and Pressure Head

A mass of fluid acquires pressure energy when it is in contact with other masses having some form of energy. Pressure energy therefore is an energy transmitted to the fluid by another mass that possesses some energy.

$$\text{Pressure energy} = W\frac{p}{\gamma}$$

$$\text{Pressure head} = \frac{\text{Pressure energy}}{W} = \frac{p}{W}$$

Energy and Head of Flow

Total Energy of Flow, E

The total energy or head in a fluid is the sum of kinetic and potential energies. Recall that potential energies are pressure energy and elevation energy.

Total energy = Kinetic energy + Pressure energy + Elevation energy

Total head = Velocity head + Pressure head + Elevation head

Fundamentals of Fluid Flow

Energy and Head of Flow

Total Energy of Flow, E

In symbol, the total head is

$$E = \frac{v^2}{2g} + \frac{p}{\gamma} + z$$

Where:

v = mean velocity of flow (m/sec in SI and ft/sec in English)

p = fluid pressure (N/m^2 or Pa in SI and lb/ft^2 or psf in English)

z = position of fluid above or below the datum plane (m in SI and ft in English)

g = gravitational acceleration (9.81 m/sec^2 in SI and 32.2 ft/sec^2 in English)

γ = Unit weight of fluid (N/m^3 in SI and lb/ft^3 in English)

Energy and Head of Flow

Power and Efficiency

Power is the rate of doing work per unit of time. For a fluid of unit weight γ (N/m^3) flowing at the rate of Q (m^3/sec) with a total energy of E (m), the power ($Watt$) is,

$$\text{Power} = Q\gamma E$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100\%$$

Note:

1 horsepower (hp) = 746 Watts

1 horsepower (hp) = 550 ft-lb/sec

1 Watt = 1 N-m/sec = 1 Joule/sec

Fundamentals of Fluid Flow

Bernoulli's Energy Theorem

Applying the law of conservation of energy to fluids that may be considered incompressible, Bernoulli's theorem may be stated as follows:

Neglecting head lost, the total amount of energy per unit weight is constant at any point in the path of flow.

Daniel Bernoulli
(1700 - 1782)

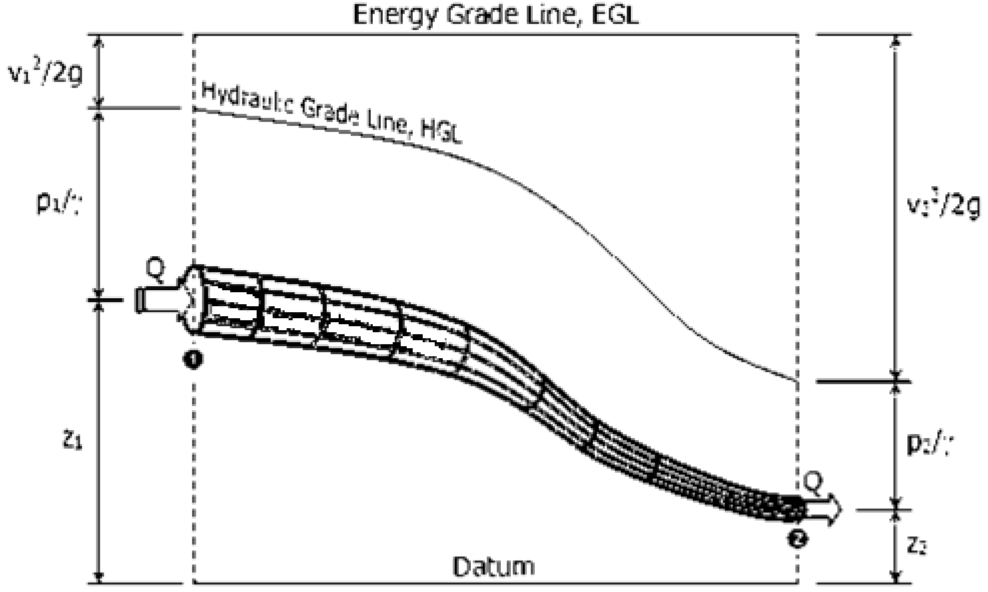
Bernoulli's Energy Equations

Energy Equation Neglecting Head Loss

Without head losses, the total energy at point (*1*) is equal to the total energy at point (*2*). No head lost is an ideal condition leading to theoretical values in the results.

$$E_1 = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$



Fundamentals of Fluid Flow

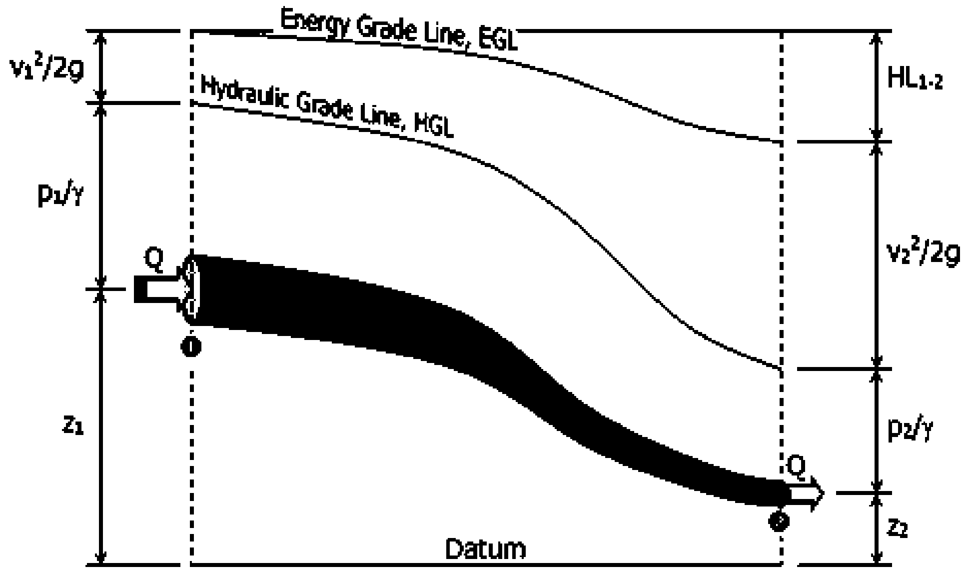
Bernoulli's Energy Equations

Energy Equation Considering Head Loss

The actual values can be found by considering head losses in the computation of flow energy.

$$E_1 - HL_{1-2} = E_2$$

$$\frac{v_1{}^2}{2g} + \frac{p_1}{\gamma} + z_1 - HL_{1-2} = \frac{v_2{}^2}{2g} + \frac{p_2}{\gamma} + z_2$$



Fundamentals of Fluid Flow

Bernoulli's Energy Equations

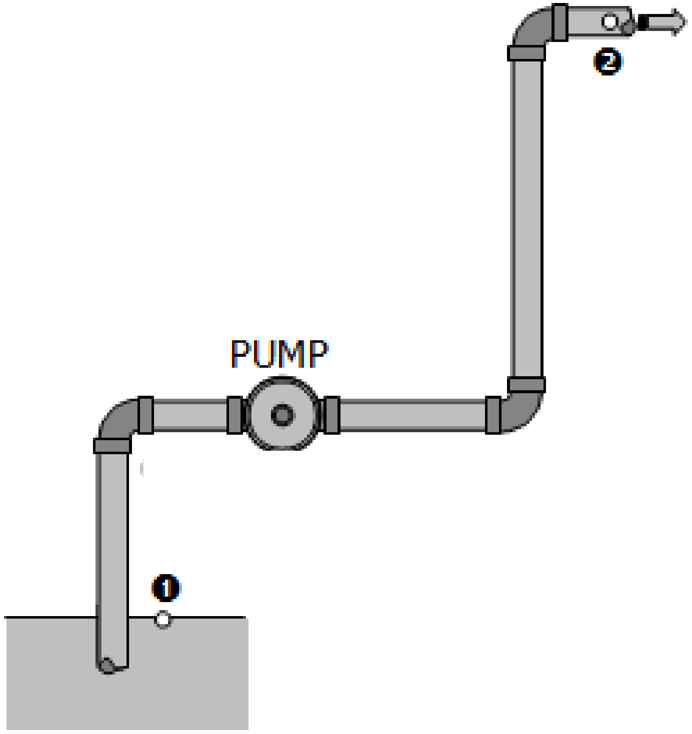
Energy Equation With Pump

In most cases, pump is used to raise water from lower elevation to higher elevation. In a more technical term, the use of pump is basically to increase the energy of flow. The pump consumes electrical energy (P_{input}) and delivers flow energy (P_{output}).

$$E_1 + HA - HL_{1-2} = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 + HA - HL_{1-2} = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\text{Output power of pump} = Q\gamma HA$$

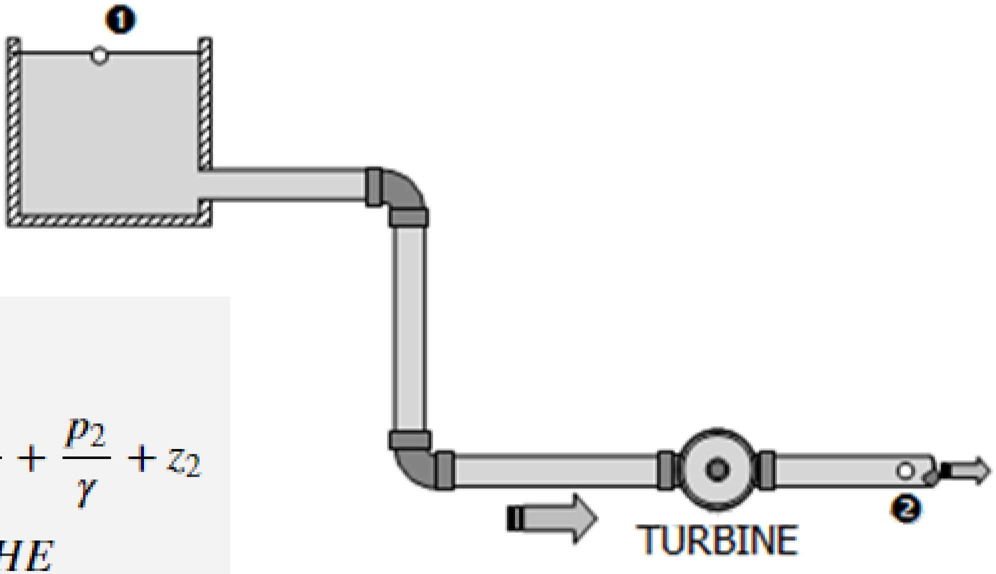


Fundamentals of Fluid Flow

Bernoulli's Energy Equations

Energy Equation With Turbine

Turbines extract flow energy and converted it into mechanical energy which in turn converted into electrical energy.



$$E_1 - HE - HL_{1-2} = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - HE - HL_{1-2} = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\text{Input power of turbine} = Q\gamma HE$$

Hydraulic and Energy Grade Lines

Hydraulic Grade Line (HGL)

Hydraulic grade line, also called hydraulic gradient and pressure gradient, is the graphical representation of the potential head (*pressure head + elevation head*). It is the line to which liquid rises in successive piezometer tubes. The line is always at a distance ($p/\gamma + z$) above the datum plane.

Characteristics of HGL

- HGL slopes downward in the direction of flow but it may rise or fall due to change in pressure.
- HGL is parallel to EGL for uniform pipe cross section.
- For horizontal pipes with constant cross section, the drop in pressure gradient between two points is equivalent to the head lost between these points.

Hydraulic and Energy Grade Lines

Energy Grade Line (EGL)

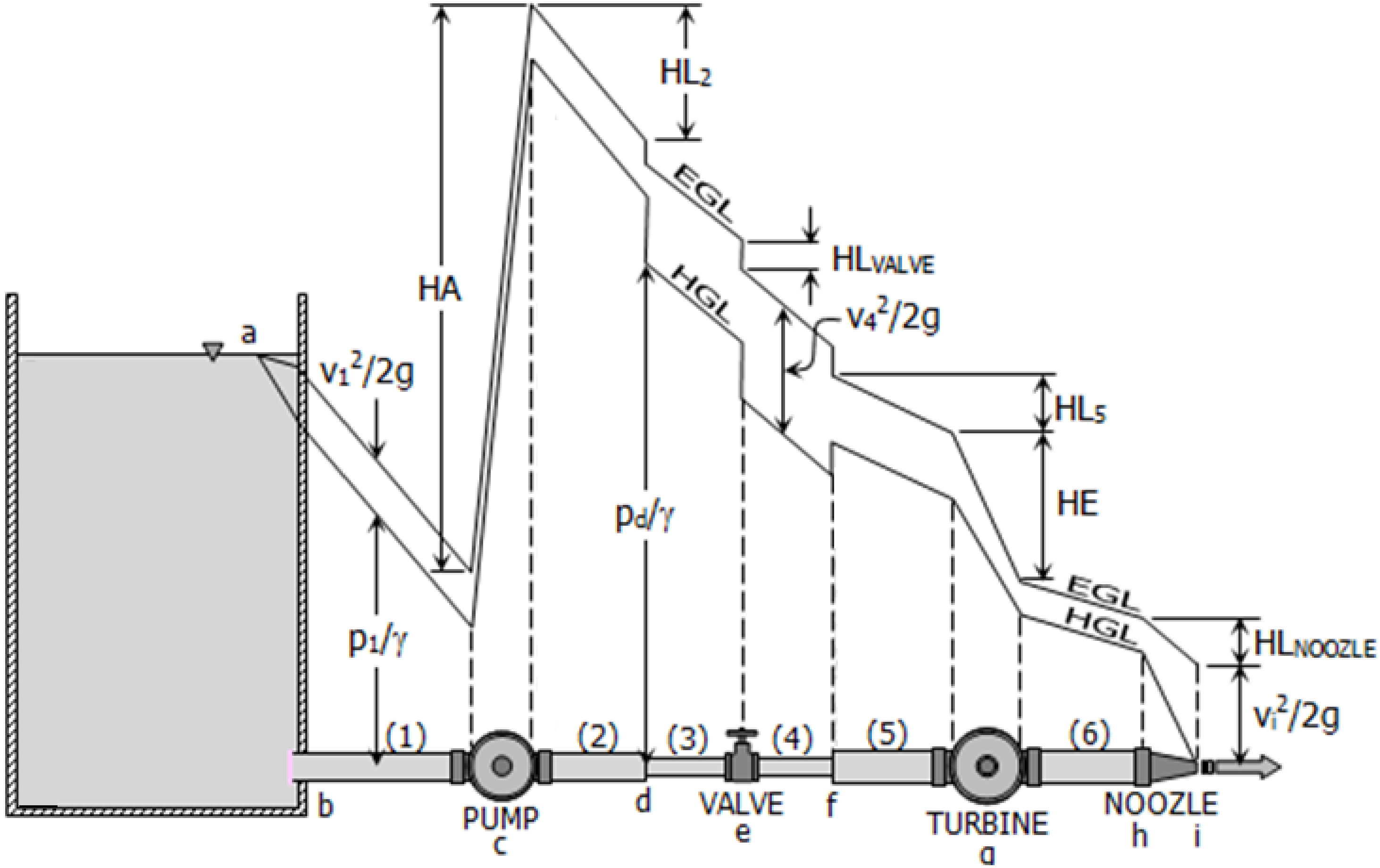
Energy grade line is always above the hydraulic grade line by an amount equal to the velocity head. Thus, the distance of energy gradient above the datum plane is always ($v^2/2g + p/\gamma + z$). Energy grade line therefore is the graphical representation of the total energy of flow.

Characteristics of EGL

- EGL slopes downward in the direction of flow and will only rise with the presence of pump.
- The vertical drop of EGL between two points is the head lost between those points.
- EGL is parallel to HGL for uniform pipe cross section.
- EGL is always above the HGL by $v^2/2g$.
- Neglecting head loss, EGL is horizontal.

Fundamentals of Fluid Flow

Hydraulic and Energy Grade Lines



Problem Set 13

Problem 1

The water surface shown in Figure 13.1 is 6 *m* above the datum. The pipe is 150 *mm* in diameter and the total loss of head between point (*1*) in the water surface and point (*5*) in the jet is 3 *m*. Determine the velocity of flow in the pipe and the discharge *Q*.

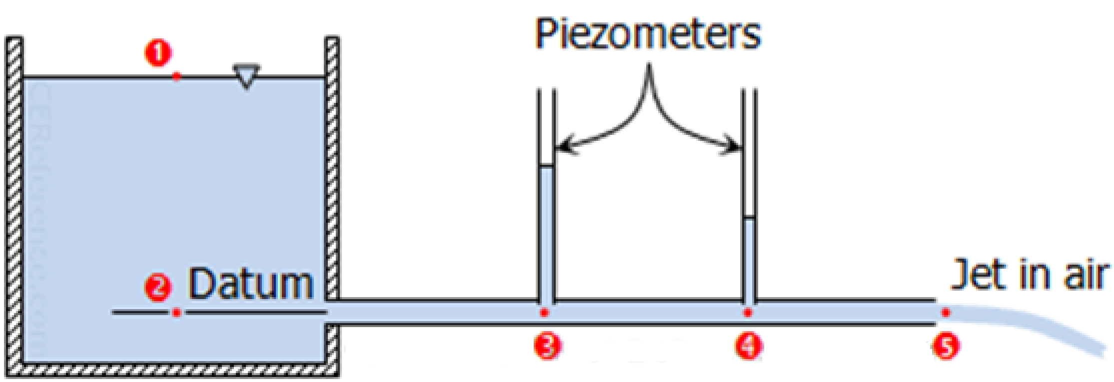


Fig. 13.1

Problem Set 13

Problem 1

Solution:

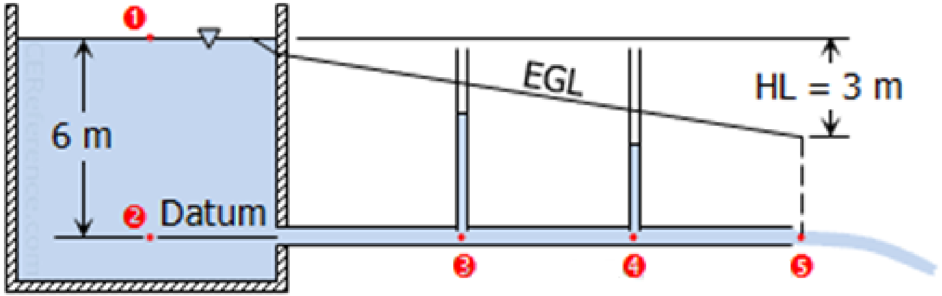
Solve for velocity head at point (5)

$$E_1 - HL_{1-5} = E_5$$

$$\left(\frac{v_1^{\,2}}{2g} + \frac{p_1}{\gamma} + z_1\right) - HL_{1-5} = \left(\frac{v_5^{\,2}}{2g} + \frac{p_5}{\gamma} + z_5\right)$$

$$(0 + 0 + 6) - 3 = \left(\frac{v_5^{\,2}}{2g} + 0 + 0\right)$$

$$\frac{v_5^{\,2}}{2g} = 3 \text{ m}$$



Fundamentals of Fluid Flow

Problem Set 13

Problem 1

Velocity of flow

$$v_5 = \sqrt{2g(3)} = \sqrt{2(9.81)(3)}$$

$$v_5 = 7.672 \text{ m/sec}$$

Since the diameter of the water jet at point (5) is equal to the diameter of the pipe

$$v_{pipe} = v_5$$

$$v_{pipe} = 7.672 \text{ m/sec} \qquad \textit{answer}$$

Discharge

$$Q = vA = 7.672[\tfrac{1}{4}\pi(0.150^2)]$$

$$Q = 0.1356 \text{ m}^3\text{/sec}$$

$$Q = 135.6 \text{ Lit/sec} \qquad \textit{answer}$$

Fundamentals of Fluid Flow

Problem Set 13

Problem 2

From Figure 13.2, the following head losses are known: From (*1*) to (*2*), 0 *m*; from (*2*) to (*3*), 0.60 *m*; from (*3*) to (*4*), 2.1 *m*; from (*4*) to (*5*), 0.3 *m*. Make a table showing elevation head, velocity head, pressure head, and total head at each of the five points. How high above the center of the pipe will water stands in the piezometer tubes (*3*) and (*4*)?

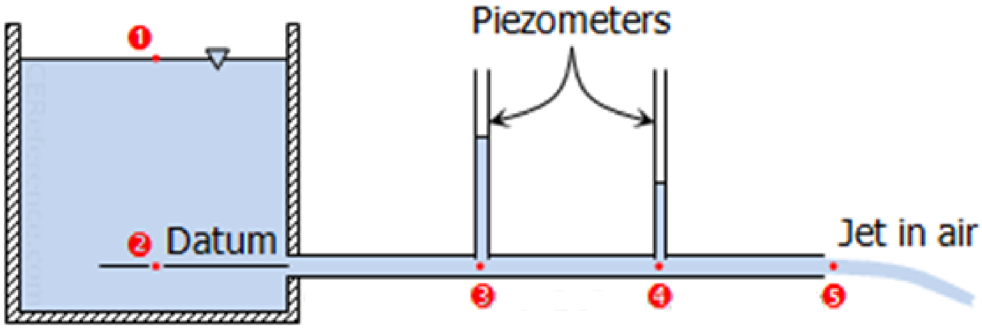


Fig. 13.2

Fundamentals of Fluid Flow

Problem Set 13

Problem 2

Solution:

Given:

$HL_{1-2} = 0$

$HL_{2-3} = 0.6 \text{ m}$

$HL_{3-4} = 2.1 \text{ m}$

$HL_{4-5} = 0.3 \text{ m}$

Total head loss from (1) to (5)

$HL_{1-5} = HL_{1-2} + HL_{2-3} + HL_{3-4} + HL_{4-5}$

$HL_{1-5} = 0 + 0.6 + 2.1 + 0.3$

$HL_{1-5} = 3 \text{ m}$

Note:

$$E = \frac{v^2}{2g} + \frac{p}{\gamma} + z$$

Sum up head from (1) to (5)

$E_1 - HL_{1-5} = E_5$

$$(0 + 0 + 6) - 3 = \left(\frac{v_5{}^2}{2g} + 0 + 0\right)$$

$$\frac{v_5{}^2}{2g} = 3 \text{ m}$$

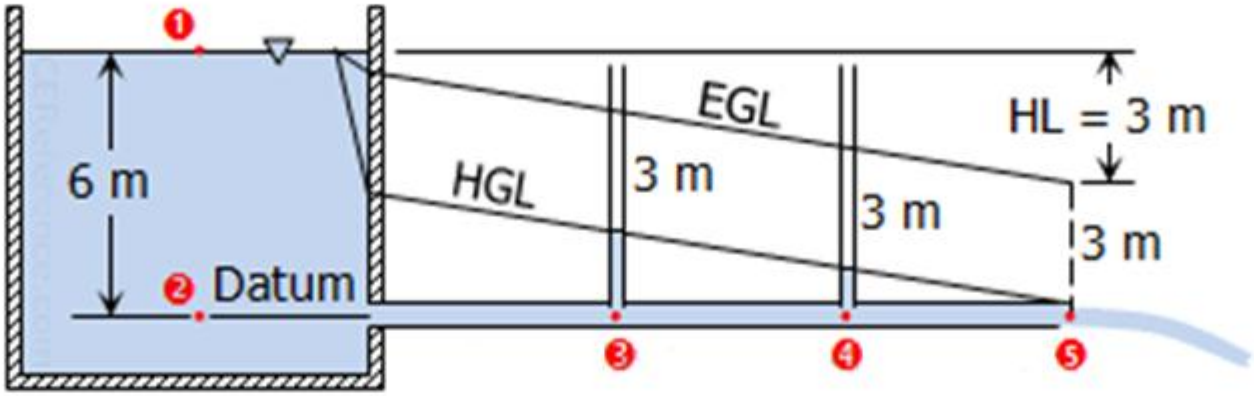
Fundamentals of Fluid Flow

Problem Set 13

Problem 2

Since the diameter of the pipe is uniform and the opening for the jet is equal to the diameter of the pipe, the velocity heads at any point on the pipe are equal. Thus,

$$\frac{v_3^2}{2g} = \frac{v_4^2}{2g} = \frac{v_5^2}{2g} = 3\ \text{m}$$



Fundamentals of Fluid Flow

Problem Set 13

Problem 2

Sum up head from (1) to (2)

$$E_1 - HL_{1-2} = E_2$$

$$(0+0+6) - 0 = \left(0 + \frac{p_2}{\gamma} + 0\right)$$

$$\frac{p_2}{\gamma} = 6 \text{ m}$$

Sum up head from (2) to (3)

$$E_2 - HL_{2-3} = E_3$$

$$(0+6+0) - 0.6 = \left(3 + \frac{p_3}{\gamma} + 0\right)$$

$$\frac{p_3}{\gamma} = 2.4 \text{ m}$$

Sum up head from (3) to (4)

$$E_3 - HL_{3-4} = E_4$$

$$(3+2.4+0) - 2.1 = \left(3 + \frac{p_4}{\gamma} + 0\right)$$

$$\frac{p_4}{\gamma} = 0.3 \text{ m}$$

Sum up head from (4) to (5)

$$E_4 - HL_{4-5} = E_5$$

$$(3+0.3+0) - 0.3 = (3+0+0)$$

$$3 = 3 \text{ (check)}$$

Fundamentals of Fluid Flow

Problem Set 13

Problem 2

Tabulated result

Point	Elevation head (m)	Velocity head (m)	Pressure head (m)	Total head (m)
1	6	0	0	6.0
2	0	0	6	6.0
3	0	3	2.4	5.4
4	0	3	0.3	3.3
5	0	3	0	3.0

Problem Set 13

Problem 2

Piezometric heights

$$h = \frac{p}{\gamma} + z$$

$h_3 = 0 + 2.4 = 2.4 \text{ m}$ *answer*

$h_4 = 0 + 0.3 = 0.3 \text{ m}$ *answer*

Problem Set 13

Problem 3

A 300-*mm* pipe is connected by a reducer to a 100-*mm* pipe. See Figure 13.3. Points *1* and *2* are at the same elevation, the pressure at 1 is 200 *kPa*. The discharge *Q* is 30 *L/s* flowing from *1* to *2* and the energy lost from *1* to *2* is equivalent to 20 *kPa*.

3.1 Compute the pressure at 2 if the liquid is water.

3.2 Compute the pressure at 2 if the liquid is oil (*s* = 0.80).

3.3 Compute the pressure at 2 if the liquid is molasses (*s* = 1.5).

Problem Set 13

Problem 3

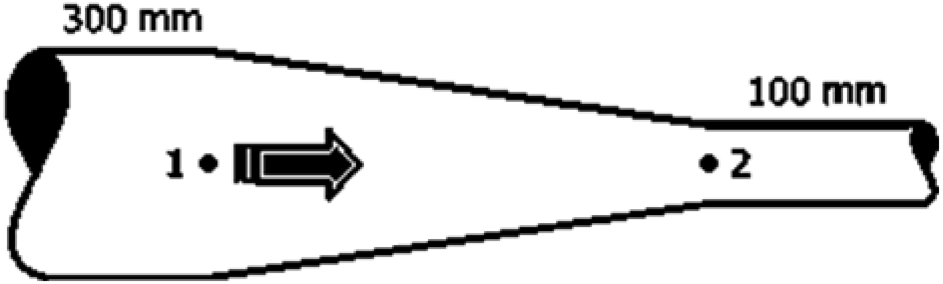


Fig. 13.3

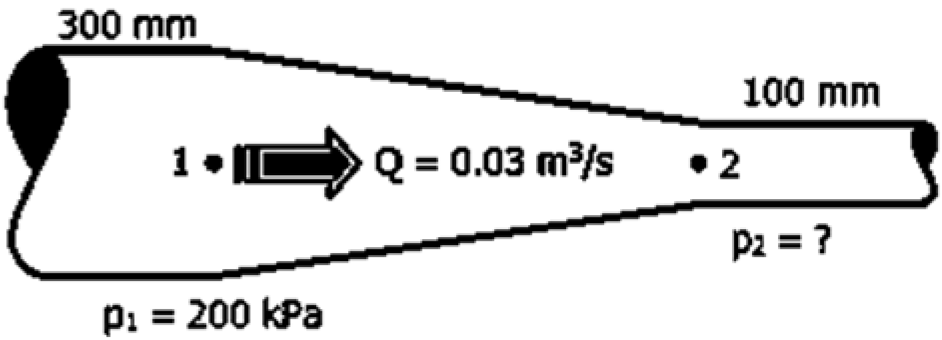
Solution:

Discharge

$$Q_1 = Q_2 = Q = 0.03 \ \text{m}^3/\text{s}$$

Head loss

$$HL = \frac{20}{\gamma} \ \text{m}$$



Problem Set 13

Problem 3

Velocity heads

$$\frac{v^2}{2g} = \frac{8Q^2}{\pi^2 g D^4}$$

$$\frac{v_1{}^2}{2g} = \frac{8(0.03^2)}{\pi^2(9.81)(0.3^4)} = 0.0092 \text{ m}$$

$$\frac{v_2{}^2}{2g} = \frac{8(0.03^2)}{\pi^2(9.81)(0.1^4)} = 0.7436 \text{ m}$$

Energy equation between 1 and 2

$$E_1 - HL = E_2$$

$$\frac{v_1{}^2}{2g} + \frac{p_1}{\gamma} + z_1 - HL = \frac{v_2{}^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$0.0092 + \frac{200}{\gamma} + 0 - \frac{20}{\gamma} = 0.7436 + \frac{p_2}{\gamma} + 0$$

$$\frac{p_2}{\gamma} = \frac{180}{\gamma} - 0.7344$$

$$p_2 = 180 - 0.7344\gamma$$

Problem Set 13

Problem 3

Part a: The liquid is water:

$$p_2 = 180 - 0.7344(9.81)$$

$$p_2 = 172.79 \text{ kPa} \qquad \textit{answer}$$

Part b: The liquid is oil (sp gr = 0.80):

$$p_2 = 180 - 0.7344(0.80 \times 9.81)$$

$$p_2 = 174.24 \text{ kPa} \qquad \textit{answer}$$

Part 3: The liquid is molasses (s = 1.5):

$$p_2 = 180 - 0.7344(1.5 \times 9.81)$$

$$p_2 = 169.19 \text{ kPa} \qquad \textit{answer}$$

Problem Set 13

Problem 4

The diameter of a pipe carrying water changes gradually from 150 *mm* at *A* to 450 *mm* at *B*. *A* is 4.5 *m* lower than *B*. If the pressure at *A* is 70 *kPa* and that *B* is 50 *kPa*, when 140 *L/s* is flowing.

4.1 Determine the direction of flow.

4.2 Find the frictional loss between the two points.

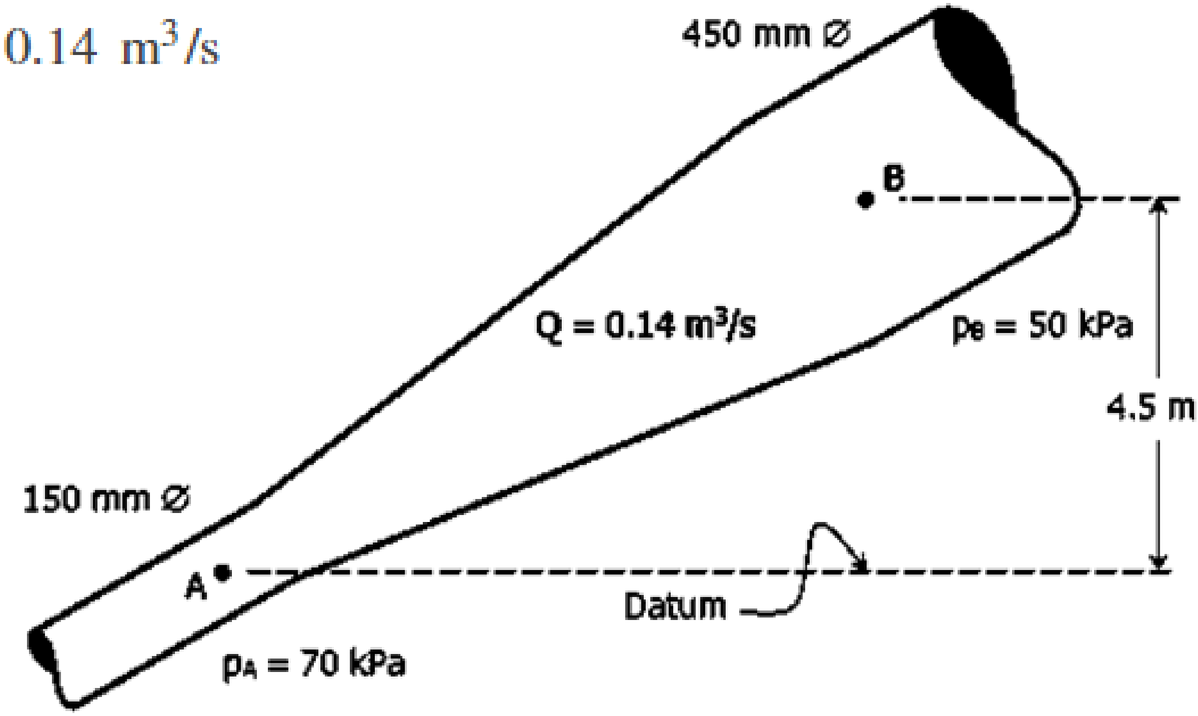
Fundamentals of Fluid Flow

Problem Set 13

Problem 4

Solution:

$$Q_A = Q_B = 0.14 \ \text{m}^3/\text{s}$$



Fundamentals of Fluid Flow

Problem Set 13

Problem 4

Velocity heads

$$\frac{v^2}{2g} = \frac{8Q^2}{\pi^2 g D^4}$$

$$\frac{v_A{}^2}{2g} = \frac{8(0.14^2)}{\pi^2(9.81)(0.15^4)} = 3.1990 \text{ m}$$

$$\frac{v_B{}^2}{2g} = \frac{8(0.14^2)}{\pi^2(9.81)(0.45^4)} = 0.0395 \text{ m}$$

Pressure heads

$$\frac{p_A}{\gamma} = \frac{70}{9.81} = 7.1356 \text{ m}$$

$$\frac{p_B}{\gamma} = \frac{50}{9.81} = 5.0968 \text{ m}$$

Total head

$$E = \frac{v^2}{2g} + \frac{p}{\gamma} + z$$

$$E_A = 3.1990 + 7.1356 + 0 = 10.3346 \text{ m}$$

$$E_B = 0.0395 + 5.0968 + 4.5 = 9.6363 \text{ m}$$

Problem Set 13

Problem 4

The flow is always from higher energy to lower energy. $E_A > E_B$, thus, the flow will be from A to B. *answer*

Energy equation between A and B

$$E_A - HL = E_B$$

$$10.3346 - HL = 9.6363$$

$$HL = 0.6983 \text{ m}$$ *answer*

Problem Set 13

Problem 5

A pump (Figure 13.5) takes water from a 200-*mm* suction pipe and delivers it to a 150-*mm* discharge pipe in which the velocity is 3.6 *m/s*. The pressure is -35 *kPa* at *A* in the suction pipe. The 150-*mm* pipe discharges horizontally into air at *C*. To what height *h* above *B* can the water be raised if *B* is 1.8 *m* above *A* and 20 *hp* is delivered to the pump? Assume that the pump operates at 70% efficiency and that the frictional loss in the pipe between *A* and *C* is 3 *m*.

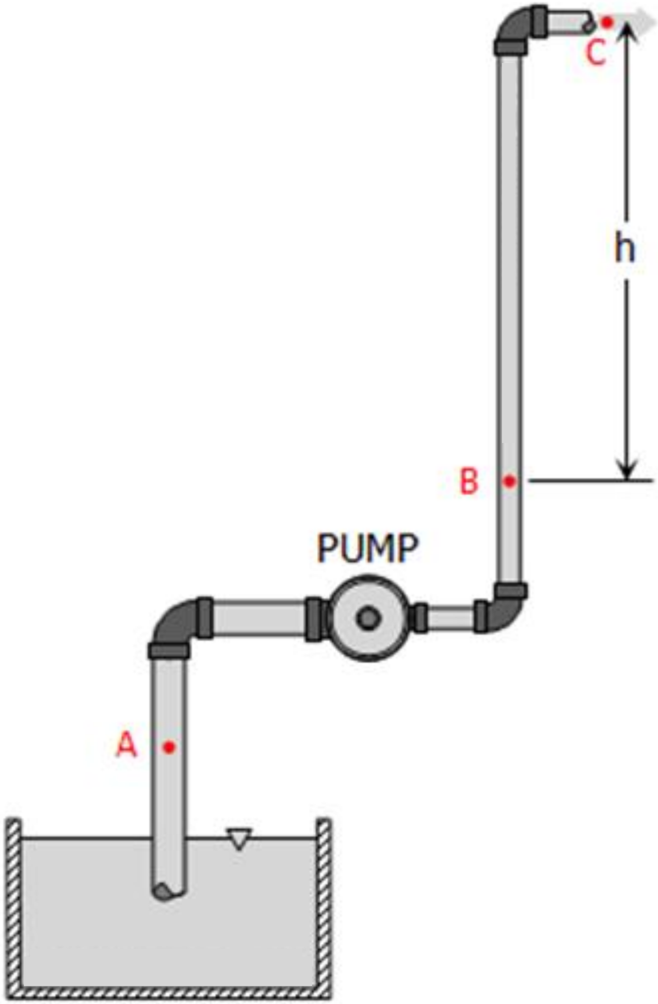


Fig. 13.5

Problem Set 13

Problem 5

Solution:

Discharge

$$Q = v_B A_B$$

$$Q = 3.6\left[\tfrac{1}{4}\pi(0.15^2)\right]$$

$$Q = 0.0636 \ \text{m}^3/\text{s}$$

Output power of the pump

$$P_{output} = \text{Efficiency} \times P_{input}$$

$$P_{output} = 0.70 P_{input} = 0.70(20)$$

$$P_{output} = 14 \text{ hp}(746 \text{ Watts}/1 \text{ hp})$$

$$P_{output} = 10\,444 \ \text{Watts}$$

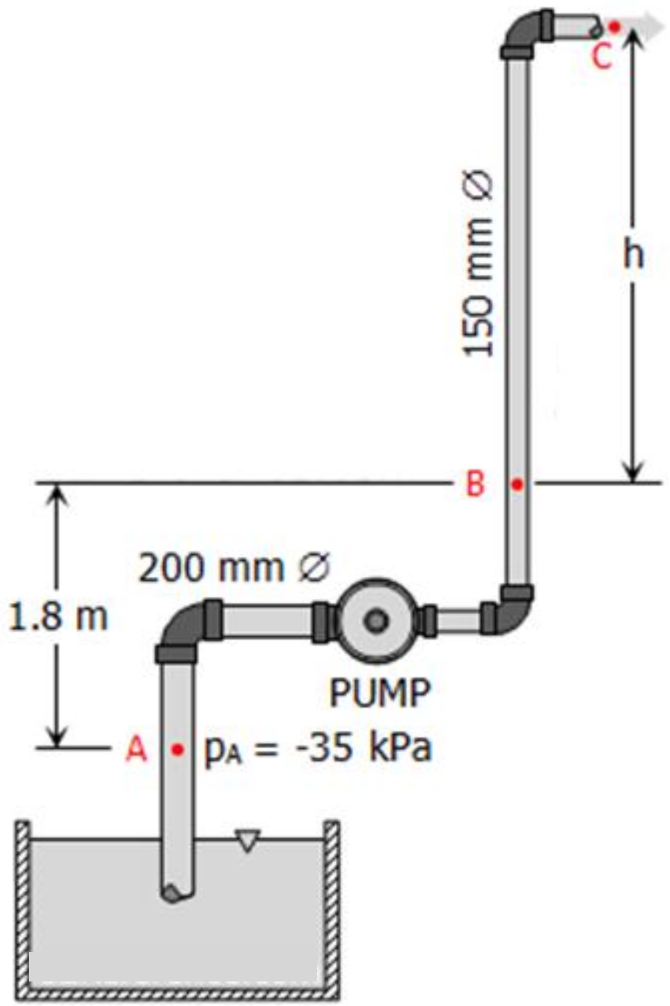


Fig. 13.5

Fundamentals of Fluid Flow

Problem Set 13

Problem 5

Head Added

$$P_{output} = Q\gamma HA$$

$$10\,444 = 0.0636(9810)HA$$

$$HA = 16.74 \text{ m}$$

Velocity heads

$$\frac{v^2}{2g} = \frac{8Q^2}{\pi^2 g D^4}$$

$$\frac{v_A^{\,2}}{2g} = \frac{8(0.0636^2)}{\pi^2(9.81)(0.2^4)} = 0.2089 \text{ m}$$

$$\frac{v_B^{\,2}}{2g} = \frac{8(0.0636^2)}{\pi^2(9.81)(0.15^4)} = 0.6602 \text{ m}$$

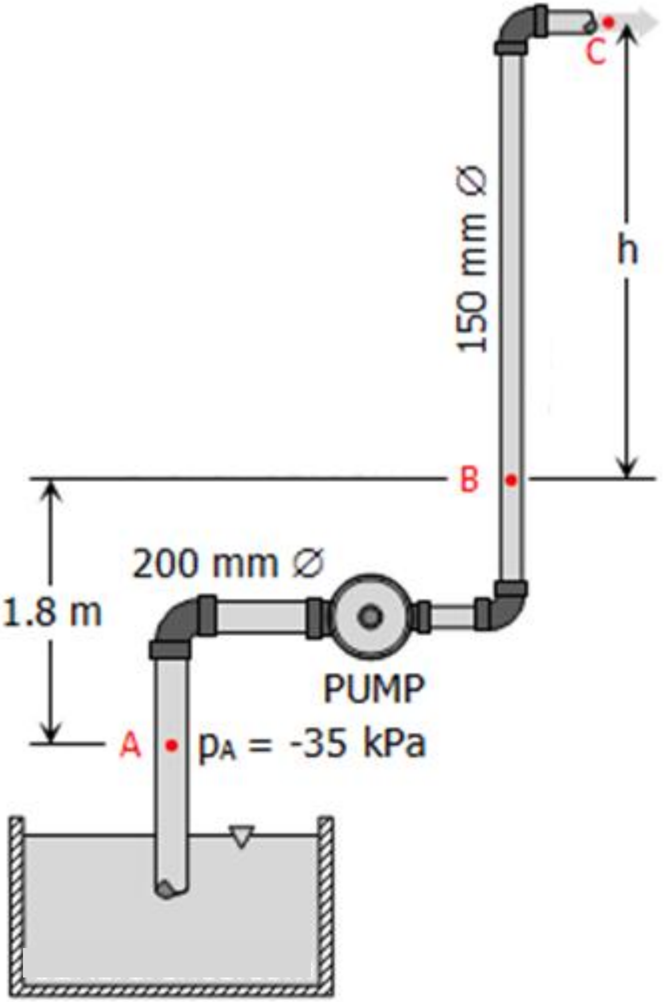


Fig. 13.5

Fundamentals of Fluid Flow

Problem Set 13

Problem 5

Energy equation from A to C

$$E_A + HA - HL = E_C$$

$$\frac{v_A{}^2}{2g} + \frac{p_A}{\gamma_w} + z_A + HA - HL = \frac{v_C{}^2}{2g} + \frac{p_C}{\gamma} + z_C$$

$$0.2089 - \frac{35}{9.81} + 0 + 16.74 - 3 = 0.6602 + 0 + (1.8 + h)$$

$h = 7.92$ m *answer*

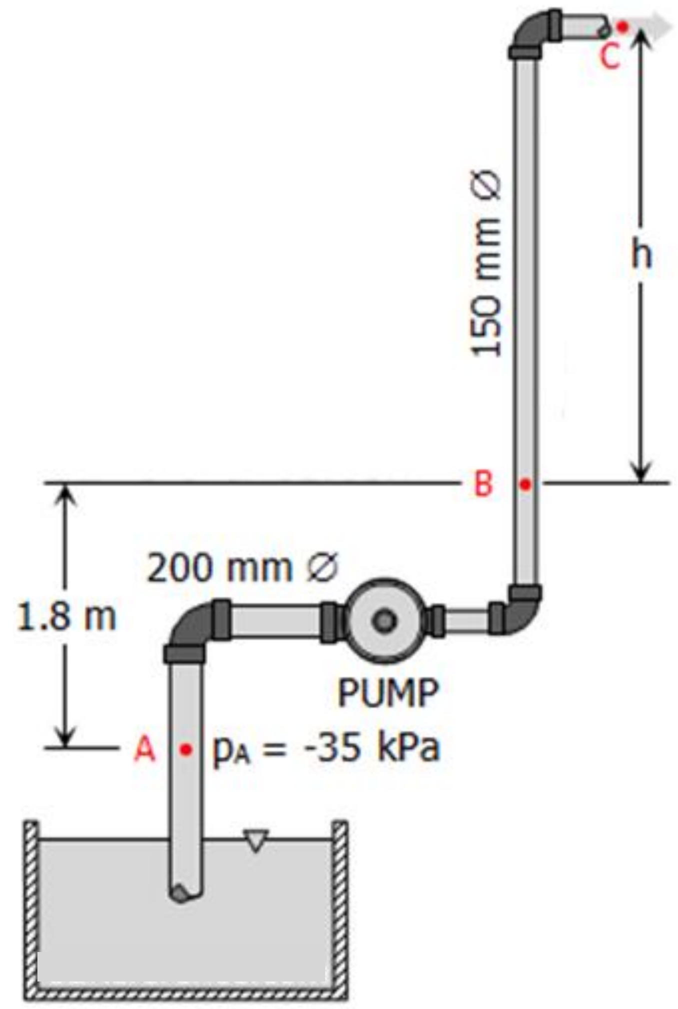


Fig. 13.5

Problem Set 13

Problem 6

A pump draws water from reservoir *A* and lifts it to reservoir *B* as shown in Figure 13.6. The loss of head from *A* to *1* is 3 times the velocity head in the 150-*mm* pipe and the loss of head from *2* to *B* is 20 times the velocity head in the 100-*mm* pipe. Compute the horsepower output of the pump and the pressure heads at *1* and *2* when the discharge is: (*a*) 12 *L/s*; (*b*) 36 *L/s*.

Fundamentals of Fluid Flow

Problem Set 13

Problem 6

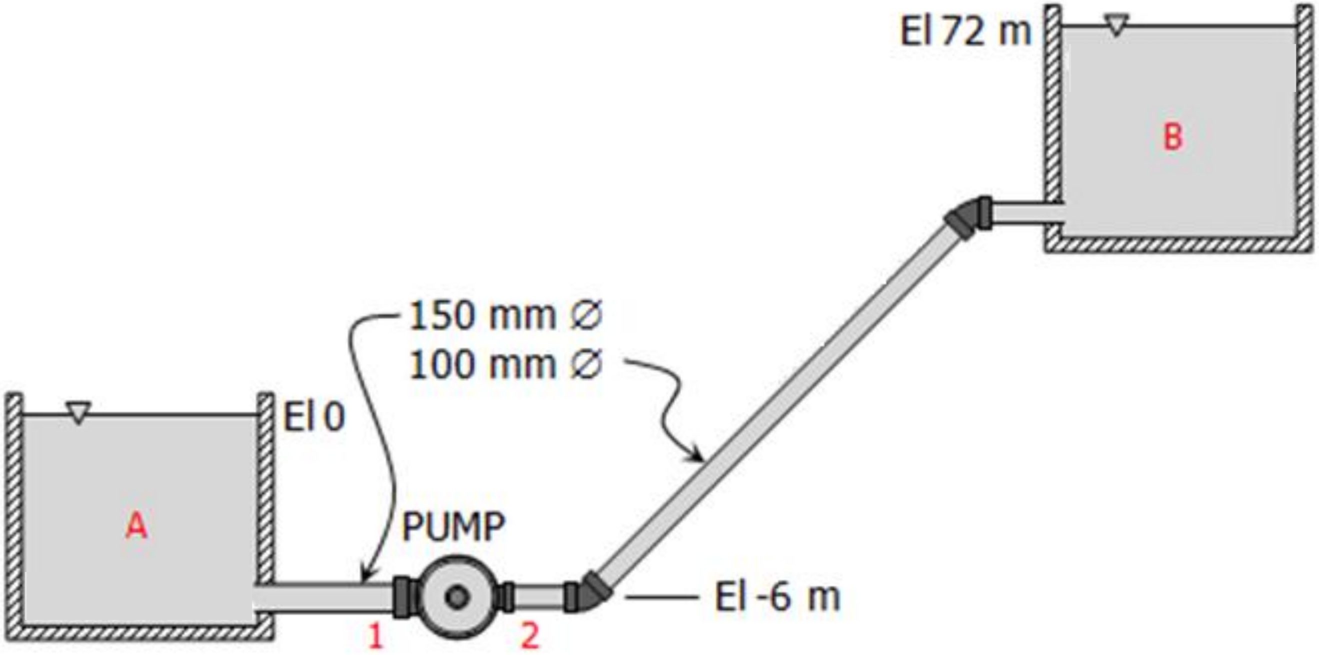


Fig. 13.6

Problem Set 13

Problem 6

Solution:

Part (a): Discharge, Q = 12 L/s

$Q = 0.012 \ \text{m}^3/\text{s}$

Velocity heads

$$\frac{v^2}{2g} = \frac{8Q^2}{\pi^2 g D^4}$$

$$\frac{v_1^2}{2g} = \frac{8(0.012^2)}{\pi^2(9.81)(0.15^4)} = 0.0235 \ \text{m}$$

$$\frac{v_2^2}{2g} = \frac{8(0.012^2)}{\pi^2(9.81)(0.1^4)} = 0.119 \ \text{m}$$

Head lost

$$HL_{A-1} = 3 \times \frac{v_1^2}{2g} = 3(0.0235) = 0.0705 \ \text{m}$$

$$HL_{2-B} = 20 \times \frac{v_2^2}{2g} = 20(0.119) = 2.38 \ \text{m}$$

Energy equation between A and B

$$E_A - HL_{A-1} + HA - HL_{2-B} = E_B$$

$$\left(\frac{v_A^2}{2g} + \frac{p_A}{\gamma} + z_A\right) - HL_{A-1} + HA - HL_{2-B} = \left(\frac{v_B^2}{2g} + \frac{p_B}{\gamma} + z_B\right)$$

$$(0 + 0 + 0) - 0.0705 + HA - 2.38 = (0 + 0 + 72)$$

$$HA = 74.4505 \ \text{m}$$

Problem Set 13

Problem 6

Output power of the pump

$$P = Q\gamma HA = 0.012(9810)(74.4505)$$

$$P = 8764.31\text{Watts} \times (1\text{hp}/746\text{Watts})$$

$$P = 11.75 \text{ hp} \qquad \textit{answer}$$

Energy equation between A and 1

$$E_A - HL_{A-1} = E_1$$

$$\left(\frac{v_A{}^2}{2g} + \frac{p_A}{\gamma} + z_A\right) - HL_{A-1} = \left(\frac{v_1{}^2}{2g} + \frac{p_1}{\gamma} + z_1\right)$$

$$(0 + 0 + 0) - 0.0705 = \left(0.0235 + \frac{p_1}{\gamma} - 6\right)$$

$$\frac{p_1}{\gamma} = 5.906 \text{ m} \qquad \textit{answer}$$

Energy equation between 2 and B

$$E_2 - HL_{2-B} = E_B$$

$$\left(\frac{v_2{}^2}{2g} + \frac{p_2}{\gamma} + z_2\right) - HL_{2-B} = \left(\frac{v_B{}^2}{2g} + \frac{p_B}{\gamma} + z_B\right)$$

$$\left(0.119 + \frac{p_2}{\gamma} - 6\right) - 2.38 = (0 + 0 + 72)$$

$$\frac{p_2}{\gamma} = 80.261 \text{ m} \qquad \textit{answer}$$

Checking

Energy equation between 1 and 2

$$E_1 + HA = E_2$$

$$\left(\frac{v_1{}^2}{2g} + \frac{p_1}{\gamma} + z_1\right) + HA = \left(\frac{v_2{}^2}{2g} + \frac{p_2}{\gamma} + z_2\right)$$

$$(0.0235 + 5.906 - 6) + 74.4505 = (0.119 + 80.261 - 6)$$

$$74.38 = 74.38 \qquad (\textit{Check!})$$

Fundamentals of Fluid Flow

Problem Set 13

Problem 6

Part (b): Discharge, Q = 36 L/s

$Q = 0.036\ \text{m}^3/\text{s}$

Velocity heads

$$\frac{v^2}{2g} = \frac{8Q^2}{\pi^2 g D^4}$$

$$\frac{{v_1}^2}{2g} = \frac{8(0.036^2)}{\pi^2(9.81)(0.15^4)} = 0.2115\ \text{m}$$

$$\frac{{v_2}^2}{2g} = \frac{8(0.036^2)}{\pi^2(9.81)(0.1^4)} = 1.0708\ \text{m}$$

Head lost

$$HL_{A-1} = 3 \times \frac{{v_1}^2}{2g} = 3(0.2115) = 0.6345\ \text{m}$$

$$HL_{2-B} = 20 \times \frac{{v_2}^2}{2g} = 20(1.0708) = 21.416\ \text{m}$$

Energy equation between A and B

$$E_A - HL_{A-1} + HA - HL_{2-B} = E_B$$

$$\left(\frac{{v_A}^2}{2g} + \frac{p_A}{\gamma} + z_A\right) - HL_{A-1} + HA - HL_{2-B} = \left(\frac{{v_B}^2}{2g} + \frac{p_B}{\gamma} + z_B\right)$$

$$(0 + 0 + 0) - 0.6345 + HA - 21.416 = (0 + 0 + 72)$$

$$HA = 94.0505\ \text{m}$$

Fundamentals of Fluid Flow

Problem Set 13

Problem 6

Output power of the pump

$$P = Q\gamma HA = 0.036(9810)(94.0505)$$

$$P = 33\ 214.87\text{Watts} \times (1\text{hp}/746\text{Watts})$$

$$P = 44.52\ \text{hp} \qquad \textit{answer}$$

Energy equation between A and 1

$$E_A - HL_{A-1} = E_1$$

$$\left(\frac{v_A{}^2}{2g} + \frac{p_A}{\gamma} + z_A\right) - HL_{A-1} = \left(\frac{v_1{}^2}{2g} + \frac{p_1}{\gamma} + z_1\right)$$

$$(0 + 0 + 0) - 0.6345 = \left(0.2115 + \frac{p_1}{\gamma} - 6\right)$$

$$\frac{p_1}{\gamma} = 5.154\ \text{m} \qquad \textit{answer}$$

Energy equation between 2 and B

$$E_2 - HL_{2-B} = E_B$$

$$\left(\frac{v_2{}^2}{2g} + \frac{p_2}{\gamma} + z_2\right) - HL_{2-B} = \left(\frac{v_B{}^2}{2g} + \frac{p_B}{\gamma} + z_B\right)$$

$$\left(1.0708 + \frac{p_2}{\gamma} - 6\right) - 21.416 = (0 + 0 + 72)$$

$$\frac{p_2}{\gamma} = 98.3452\ \text{m} \qquad \textit{answer}$$

Checking

Energy equation between 1 and 2

$$E_1 + HA = E_2$$

$$\left(\frac{v_1{}^2}{2g} + \frac{p_1}{\gamma} + z_1\right) + HA = \left(\frac{v_2{}^2}{2g} + \frac{p_2}{\gamma} + z_2\right)$$

$$(0.2115 + 5.154 - 6) + 94.0505 = (1.0708 + 98.3452 - 6)$$

$$93.416 = 93.416 \qquad \textit{(Check!)}$$

Fundamentals of Fluid Flow

Problem Set 13

Problem 7

The 600-*mm* pipe shown in Figure 13.7 conducts water from reservoir *A* to a pressure turbine, which discharges through another 600-*mm* pipe into tailrace *B*. The loss of head from *A* to *1* is 5 times the velocity head in the pipe and the loss of head from *2* to *B* is 0.2 times the velocity head in the pipe. If the discharge is 700 *L/s*, what power is being given up by the water to the turbine and what are the pressure heads at *1* and *2*?

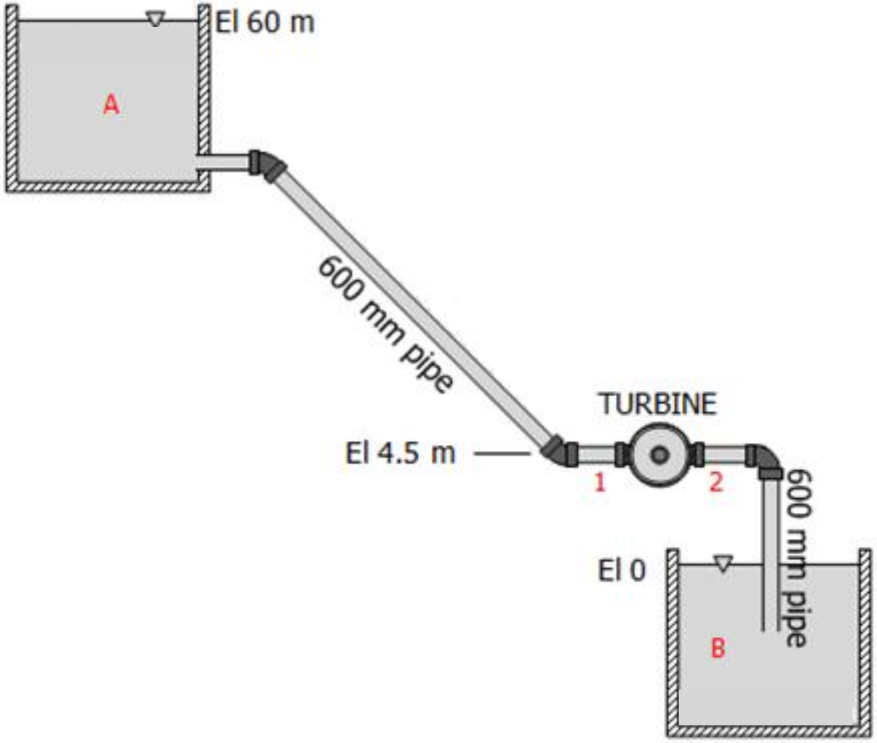


Fig. 13.7

Problem Set 13

Problem 7

Solution:

Discharge

$$Q = 0.7 \ \text{m}^3/\text{s}$$

Velocity heads

$$\frac{v^2}{2g} = \frac{8Q^2}{\pi^2 g D^4}$$

$$\frac{{v_1}^2}{2g} = \frac{{v_2}^2}{2g} = \frac{8(0.7^2)}{\pi^2(9.81)(0.6^4)} = 0.3124 \ \text{m}$$

Head lost

$$HL_{A-1} = 5 \times \frac{{v_1}^2}{2g} = 5(0.3124) = 1.562 \ \text{m}$$

$$HL_{2-B} = 0.2 \times \frac{{v_2}^2}{2g} = 0.2(0.3124) = 0.0625 \ \text{m}$$

Problem Set 13

Problem 7

Energy equation between A and B

$$E_A - HL_{A-1} - HE - HL_{2-B} = E_B$$

$$\left(\frac{v_A^2}{2g} + \frac{p_A}{\gamma} + z_A\right) - HL_{A-1} - HE - HL_{2-B} = \left(\frac{v_B^2}{2g} + \frac{p_B}{\gamma} + z_B\right)$$

$$(0 + 0 + 60) - 1.562 - HE - 0.0625 = (0 + 0 + 0)$$

$$HE = 58.3755 \text{ m}$$

Power given up by the water to the turbine

$$P = Q\gamma HE = 0.7(9810)(58.3755)$$

$$P = 400\,864.56 \text{ Watts } \times (1 \text{ hp} / 746 \text{ Watts})$$

$$P = 537.35 \text{ hp} \qquad \textit{answer}$$

Problem Set 13

Problem 7

Energy equation between A and 1

$$E_A - HL_{A-1} = E_1$$

$$\left(\frac{v_A{}^2}{2g} + \frac{p_A}{\gamma} + z_A\right) - HL_{A-1} = \left(\frac{v_1{}^2}{2g} + \frac{p_1}{\gamma} + z_1\right)$$

$$(0 + 0 + 60) - 1.562 = \left(0.3124 + \frac{p_1}{\gamma} + 4.5\right)$$

$$\frac{p_1}{\gamma} = 53.6256 \text{ m} \qquad \textit{answer}$$

Energy equation between 2 and B

$$E_2 - HL_{2-B} = E_B$$

$$\left(\frac{v_2{}^2}{2g} + \frac{p_2}{\gamma} + z_2\right) - HL_{2-B} = \left(\frac{v_B{}^2}{2g} + \frac{p_B}{\gamma} + z_B\right)$$

$$\left(0.3124 + \frac{p_2}{\gamma} + 4.5\right) - 0.0625 = (0 + 0 + 0)$$

$$\frac{p_2}{\gamma} = -4.7499 \text{ m} \qquad \textit{answer}$$

Problem Set 13

Problem 7

Checking

Energy equation between 1 and 2

$$E_1 - H_E = E_2$$

$$\left(\frac{v_1{}^2}{2g} + \frac{p_1}{\gamma} + z_1\right) - HE = \left(\frac{v_2{}^2}{2g} + \frac{p_2}{\gamma} + z_2\right)$$

$$(0.3124 + 53.6256 + 4.5) - 58.3755 = (0.3124 - 4.7499 + 4.5)$$

$$0.0625 = 0.0625 \qquad (\textit{Check!})$$